Programming in Calculators

Key Stage: 3 and 4

Strand: Measures, Shape and Space

Learning Units: Coordinate Geometry of Straight Lines (KS3)
Quadratic equations in one unknown (KS4)
Functions and graphs (KS4)

Objective: To develop computational thinking skills through programming in calculators

Pre–requisite Knowledge:
(i) using formulas of distance and slope in coordinate geometry
(ii) solving quadratic equations by the quadratic formula
(iii) relations between the discriminant of a quadratic equation and the nature of its roots
(iv) features of the graphs of quadratic functions

Relationship with other KLA(s) in STEM Education:
Computational thinking skills developed through programming in calculators are related to the learning element "Programming concepts" of Information and Communication Technology at Secondary 2.

Background information:
In the learning element "Programming concepts" of Information and Communication Technology at Secondary 2, students have to learn major stages in problem solving, including problem definition, problem analysis, algorithm design, program coding, program debugging/testing, and program documentation. In Mathematics, students come across different mathematical formulae at different key stages. It is natural for students to learn programming concepts as well as computational thinking through designing their own programs in calculators.

Description of the tasks:

Task 1: Area of triangle

The teacher uses the formula of area of triangle to introduce the basic concept of programming in calculator.

1. Students are required to find the area of triangle from the base and the height of a triangle. The teacher discusses the typical steps of programming, including "Identify the problem", "Identify the input and output", "Develop the Algorithm" and "Program".

Step	Description	Detail
1	*Identify the problem*	Find the area of triangle with given base and height.
2	*Identify the input and output*	Input: the base, the height Output: the area of the triangle
3	*Develop the algorithm*	Variables: A, B Input: the base → A, the height → B Output: A × B ÷ 2
4	*Program**	?→A: ?→B: AB⌟ 2 (12 Bytes)

* The keystrokes may vary among different calculators. The program listed here is for illustration only.

2. Students have to test whether their programs are correct by input different bases and heights and verify the outputs of their programs.

Notes for teachers:

1. The teacher has to explain to students that calculators usually have limited number of input variables for constructing programs.

2. The teacher needs to explain to students the syntax adopted by students' calculators and how the programs could be inputted to the calculators.

3. The teacher needs to remind students that, in the above example, the number of Bytes of the program is 12, in which one Byte represents one keystroke.

Task 2: Distance formula

After students have learned the formula for distance between two points in the rectangular coordinate plane, the teacher can introduce the corresponding program in calculator.

1. Students are required to complete the following table.

Step	Detail
1	*Problem*
2	*Input and Output* Input: Output:
3	*Algorithm* Variables: Input: Output:
4	*Program* (Bytes)

2. Students have to test whether their programs are correct by inputting the coordinates of different points.

Notes for teachers:

1. The teacher may ask students to suggest ways for finding the exact answer if the distance displayed on the calculator is not exact. Students may use the square function to find the exact answer.

2. Suggested answer.

Step	Detail
1	*Problem* To find the distance of two points $P(x_1, y_1)$ and $Q(x_2, y_2)$
2	*Input and Output* Input: the x-coordinates and y-coordinates of P and Q, i.e. x_1, y_1, x_2, y_2 Output: the distance of PQ
3	*Algorithm* Variables: A, B, C, D

	Input: $x_1 \to$ A, $y_1 \to$ B, $x_2 \to$ C, $y_2 \to$ D Output: $\sqrt{(A-C)^2+(B-D)^2}$
4	*Program** ?→A: ?→B: ?→C: ?→D: √((A–C)^2+(B–D)^2) (31 Bytes)

* The keystrokes may vary among different calculators. The program listed here is for illustration only.

3. The teacher may asked students to modify their programs to compute in addition the coordinates of the mid-point of P and Q, and the slope of the line passing through P and Q. Suggested answer is as follows:
?→A: ?→B: ?→C: ?→D: √((A–C)^2+(B–D)^2) ◢ (D–B) ⌟ (C–A) ◢(A+C) ⌟ 2◢ (B+D) ⌟ 2 (59 Bytes)

Task 3: Quadratic function

After students have learned the quadratic function, the teacher can introduce the program for finding the discriminant, the real root(s) if any, of a quadratic equation and the coordinates of the vertex of the corresponding quadratic function.

1. Students are required to complete the following table.

Step	Detail
1	*Problem* To find the discriminant, the real root(s) if any, of the quadratic equation $ax^2 + bx + c = 0$, and the coordinates of the vertex of the quadratic function $y = ax^2 + bx + c$.
2	*Input and Output* Input: Output:
3	*Algorithm* Variables: Input: Output:
4	*Program* (Bytes)

2. Students have to test whether their programs are correct by inputting different quadratic questions to their calculators.

Notes for teachers:

1. Suggested answer.

Step	Detail
1	*Problem* To find the discriminant, the real root(s) if any, of the quadratic equation $ax^2 + bx + c = 0$, and the coordinates of the vertex of the quadratic function $y = ax^2 + bx + c$.
2	*Input and Output* Input: the coefficient of x^2, the coefficient of x, the constant term Output: the discriminant, the real root(s) if any, and the coordinates of the vertex
3	*Algorithm* Variables: A, B, C, D Input: the coefficient of $x^2 \rightarrow$ A, the coefficient of $x \rightarrow$ B, the constant term$\rightarrow$C Output: the discriminant B^2-4AC ($\rightarrow$ D) the real root(s) if any, $(-B + \sqrt{D}) \div 2A, (-B - \sqrt{D}) \div 2A$ the coordinates of the vertex $(-B \div 2A,\ -D \div 4A)$
4	*Program** ?→A: ?→B: ?→C: B^2–4AC→D◢(–B+√(D))⌟ (2A)◢(–B–√(D))⌟ (2A)◢ –B⌟ (2A)◢–D⌟ (4A) (64 Bytes)

* The keystrokes may vary among different calculators. The program list here is for illustration only.

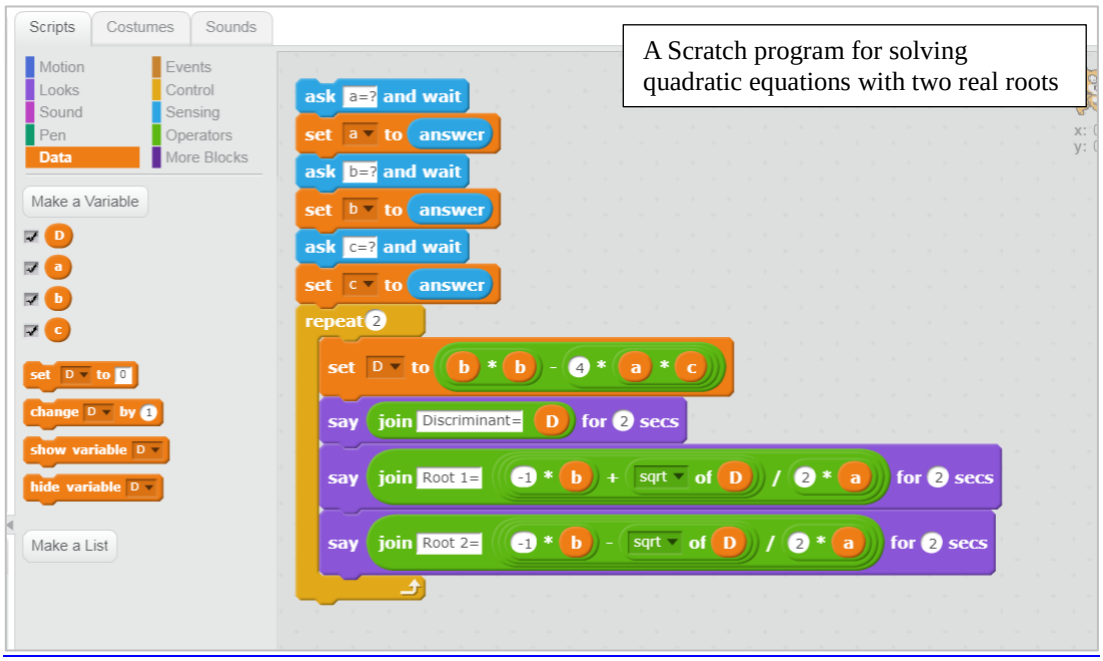
2. The teacher may ask students to check the answers for the quadratic equation $x^2 - 6x + 8 = 0$, in which the discriminant is 4, the roots are 4 and 2, and the coordinates of the vertex of $y = x^2 - 6x + 8$ are (3, –1).

3. When students try other equation, for example $x^2 - 4x + 5 = 0$, the calculator may return an error message for the roots as the discriminant is negative. Students can be asked to solve this problem. One of the ways can be
?→A: ?→B: ?→C: B^2–4AC→D◢ D<0 ⇒ Goto 0: (–B+√(D))⌟ (2A)◢ (–B–√(D))⌟ (2A)◢ Lb1 0: –B⌟ (2A)◢ –D⌟ (4A) (74 Bytes)

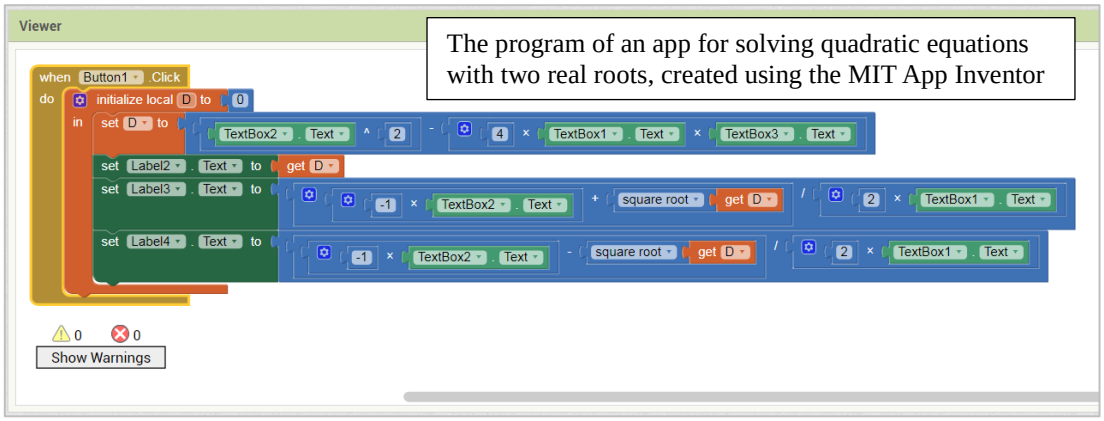
4. The teacher may also ask students to set a program for solving simultaneous linear equations in two unknowns $\begin{cases} Ap + Bq = C \\ Dp + Xq = Y \end{cases}$.
Here is a suggested answer: ?→A: ?→B: ?→C: ?→D: ?→X: ?→Y: (CX–BY)⌟ (AX–BD)→M ◢ (C–(AM)) ⌟ B (52 Bytes)

The teacher may ask students to modify the program to tackle the special case when AX–BD = 0.

5. The teacher may remind students that the computational thinking skills learnt in this activity can be transferred to set up similar algorithms for solving the same problems using other computer software, such as Scratch and MIT App Inventor.



A Scratch program for solving quadratic equations with two real roots



The program of an app for solving quadratic equations with two real roots, created using the MIT App Inventor

Reference:

1. WebCal 計數機網頁：
   http://webcal.freetzi.com/casio.fx-50FH/fx-50F_Plus.htm

2. 黃 sir 的計算機網頁：
   http://www2.hkedcity.net/sch_files/a/lpl/lpl-wwk/public_html/Casio/CasioFrame.htm